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#### ABSTRACT

If the 2D kriging interpolator is written as a function  $k(x,y)$  of the coordinates  $(x,y)$ , its expression proves equivalent to that of radial basis functions. This helps clarify the relationships between kriging and many other mapping techniques, especially splines. These relationships are discussed in the paper and many practical examples are given. Kriging with a constant trend and a De Wijs variogram ( $\gamma(h)=\text{Log}h$ ) is equivalent to 2D interpolation with harmonic splines. Thanks to its harmonic property,  $\text{Log}h$  is also associated with the familiar power spectrum of fractals. Kriging with a linear trend and a generalized covariance equal to  $h^2\text{Log}h$  is equivalent to 2D interpolation with biharmonic splines. Thanks to its biharmonic property,  $h^2\text{Log}h$  is also associated with the familiar power spectrum of fractals. Smoothing Splines calculate a function minimizing the sum of an energy functional – or regularization term – plus a distance to the data. This is equivalent to the computation of kriging with a nugget effect. In 2D, splines consist of calculating a function minimizing an energy functional, related to the stretching or a bending energy of a plate. The choice of this energy functional is equivalent to fixing the degree of the trend function and the covariance model for kriging. In other words, fixing the energy – or regularization - term of splines is equivalent to fixing the a priori model for kriging. There is a fundamental inverse relationship between the spline functional and the covariance. This fundamental relationship also applies in the frame of discrete Bayesian statistics. The consequence of this relationship on the spectral density is straightforward. In many of his writings Claerbout discusses the “geoestimation” problem. He defines the Prediction Error Filter (PEF) as the filter such that its output tends to a white spectrum. This means that the PEF is identical to that associated with the roughening spline operator derived from the inverse of the covariance. Kriging can be formalized in the frame of energy-based estimation techniques such as splines. As a result, the regularization term commonly used in inversion methods can be regarded as an expression of the prior knowledge about the phenomenon. This is due to the link between the inverse of the covariance function and the roughening filter implicit in the quadratic regularization term.